



# Controllable Physics-Aware Generative Model for Urban Travel Demand Calibration



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# Urban Travel Demand Calibration

- Transportation agencies and stakeholders worldwide commonly develop traffic simulation models of their road networks and use them to inform a variety of planning and operational decisions.
  
- Calibrating the input parameters of these simulators is an important offline optimization problem:
  - High simulation costs;
  - Specific sample from limited observed traffic information;
  - Evaluate counterfactual (what-if) scenarios
  - ...



# Motivation - Origin-Destination (OD) Calibration

- (Static) OD calibration aims to identify an OD matrix from a hypothesized distribution, resulting in simulated metrics that accurately reflect field-observed traffic conditions:
  - high-dimensionality,
  - non-convexity,
  - underdetermined,
  - simulation-based nature
  - ...
- Explore the Deep Generative Models.

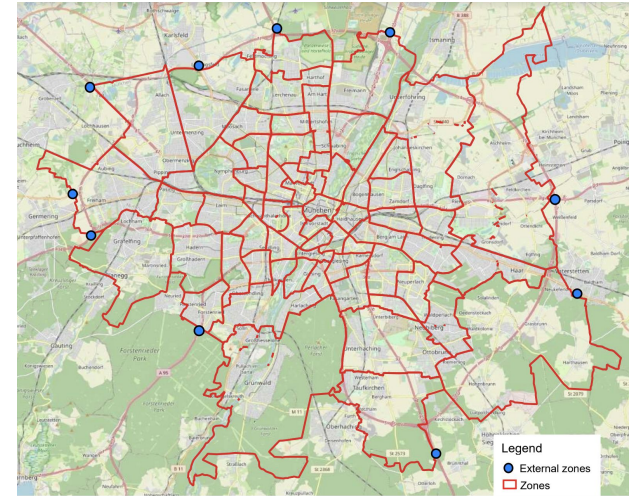


Figure 1. Topology of Munich network with major region Traffic zones

Transportation Science  
(Parameter tuning process)



Machine Learning  
(Model fitting process)



# Motivation - Gray-box / Hybrid modeling

- Gray-box: data-driven + theory-driven
- Deep Generative Models:
  - E.g., Variational Autoencoders (VAEs); Generative Adversarial Networks (GANs); Diffusion Models...
  - Extracting complex relationships via data-driven method;
  - Generating samples from the learned latent distribution.
- Physics knowledge:
  - Contributing to data efficiency;
  - Constraining the generative model's latent space;
  - Carrying out counterfactually robust transportation analysis.
  - Transportation domain: analytic equations[1] such as the continuity equations.

[1] Osorio, Carolina. "Dynamic origin-destination matrix calibration for large-scale network simulators." *Transportation Research Part C: Emerging Technologies* 98 (2019): 186-206.



# Outline

## ➤ Motivation

- Urban Travel Demand Calibration (Black box)
- Gray-box / Hybrid modeling

## ➤ Problem Setting

- Bayesian Perspective

## ➤ Controllable Physics-Aware Variational Autoencoders

- Physics analytical model
- Controllable Physics Information
- Conditional variational autoencoder (CVAE)
- Training Objective

## ➤ Results

- Experiments Settings
- Comparison to SOTA Methods
- Comparison to Neural Baselines
- Qualitative Evaluations
  - Convergence speed
  - OD Distribution
  - Traffic Flow Calibration



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# OD Calibration Problem Formulation from Bayesian Perspective

- Instead of obtaining a single calibrated point estimate  $d$  for an observed traffic flow  $y$ , we now seek a posterior  $p(d|y)$ .
- Samples  $d \sim p(d|y)$  are OD matrices likely to have yielded the observed traffic flow under the likelihood  $p_S(y|d)$  defined by the traffic simulator  $S$ .



# OD Calibration Problem Formulation from Bayesian Perspective

- The posterior can be given in the standard setup for Bayesian inference:

$$p(d|y, d^{\text{prior}}) = \frac{p(y|d)p(d|d^{\text{prior}})}{p(y|d^{\text{prior}})} = \frac{p(y|d)p(d|d^{\text{prior}})}{\int p(y|d)p(d|d^{\text{prior}})dd}$$

$p(d|d^{\text{prior}})$  defines the prior distribution, conditional on the provided noisy OD  $d^{\text{prior}}$ .

- For the likelihood, we view the traffic simulator  $S$  as implicitly defining  $p(y|d)$ :

$$p(y|d) = \int p_S(y, z|d)dz = \int p_S(y, u_1, u_2|d)du_1du_2,$$

i.e. marginalizing over the possible trajectories  $z$  in the simulator's latent space, where  $u_1, u_2$  are vectors of simulators.

➔ Conditional VAE





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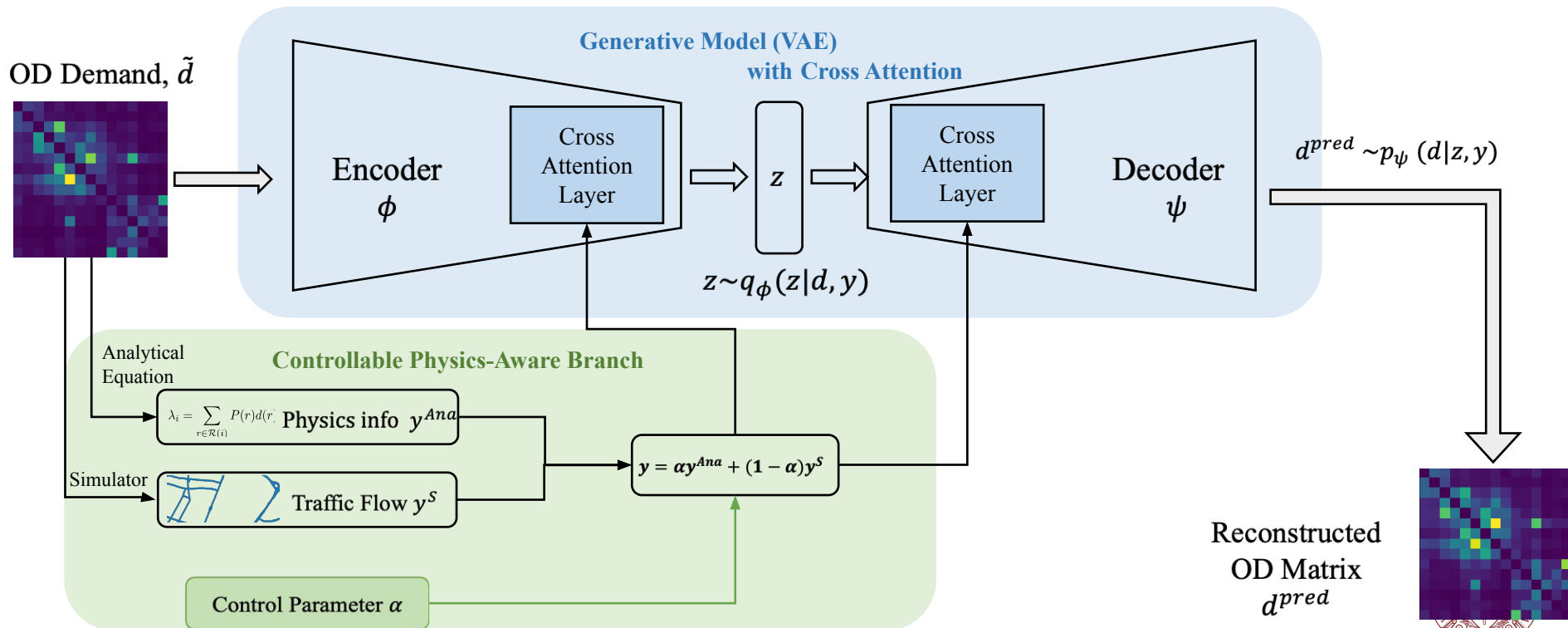
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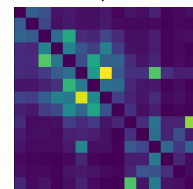
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# Controllable Physics-Aware Variational Autoencoders

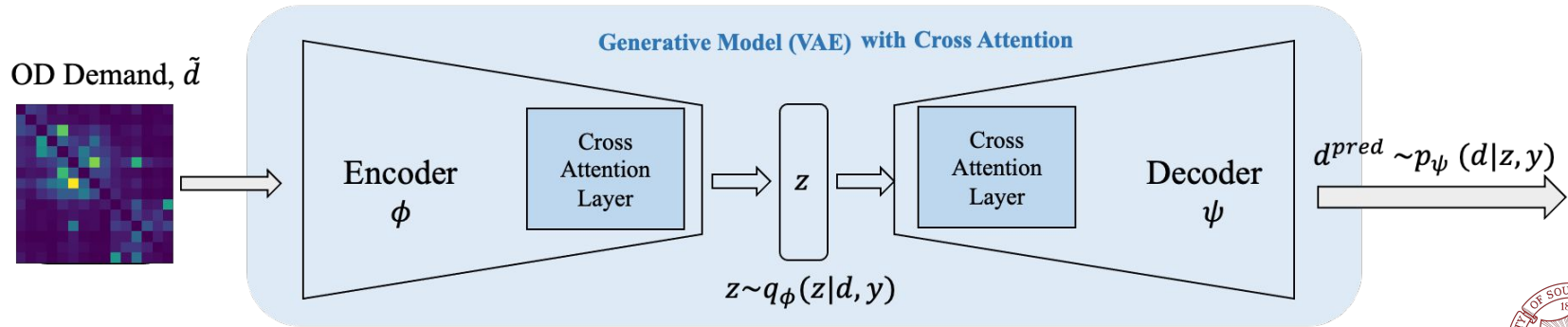


Reconstructed  
OD Matrix  
 $d^{pred}$



# Conditional variational autoencoder (CVAE)

- Encoder: learn the hidden representation of given data and the distribution of  $q_{\phi}(z|d, y)$ .
- Decoder: decode the hidden representation to input space and captures the distribution  $p_{\psi}(d|z, y)$ ;

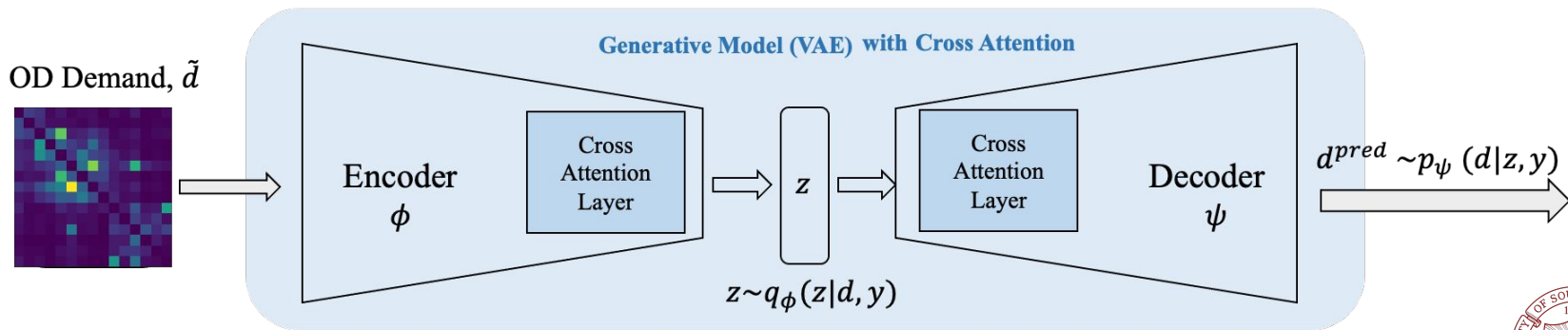


# Control-VAE

- Traffic flows and OD pairs are under the same physics mechanisms but have different modalities
- Cross-attention Fusion: traffic flows as keys and values in a cross-attention mechanism

$$\text{Att}_h(Q_h, K_h, V_h) = \text{Softmax}\left(\frac{Q_h K_h^T}{\sqrt{d_k}}\right) V_h,$$

$$\text{where } Q_h = W_h^Q f(d); K_h = W_h^K g(y); V_h = W_h^V g(y).$$



# Physics analytical model

- Linear approximation of a simulator on link  $i$  [1] based on the travel behavior:

$$\lambda_i = \sum_{r \in \mathcal{R}(i)} P(r)d(r)$$

$\mathcal{R}(i)$  denotes the set of routes that travel through link  $i$ .  $d(r)$  denotes the OD pair of route  $r$  and  $P(r)$  denotes the probability of choosing route  $r$ .

- $P(r)$  is a multinomial logit model with a utility function that depends on the route's travel time  $t_r$  [1]:

$$P(r) = \frac{\exp(\theta t_r)}{\sum_{j \in \mathcal{R}(r)} \exp(\theta t_j)},$$

$t_j$  denotes the travel time of route  $j$ ,  $\theta$  is a travel time scalar parameter.

[1] Arora, Neha, et al. "An efficient simulation-based travel demand calibration algorithm for large-scale metropolitan traffic models."

[2] Osorio, Carolina. "High-dimensional offline origin-destination (OD) demand calibration for stochastic traffic simulators of large-scale road networks." *Transportation Research Part B: Methodological* 124 (2019): 18-43.



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- **Controllable Physics Information:**
  - balance between physics-knowledge and simulation information:

$$y = \alpha y^{Ana} + (1 - \alpha) y^s.$$

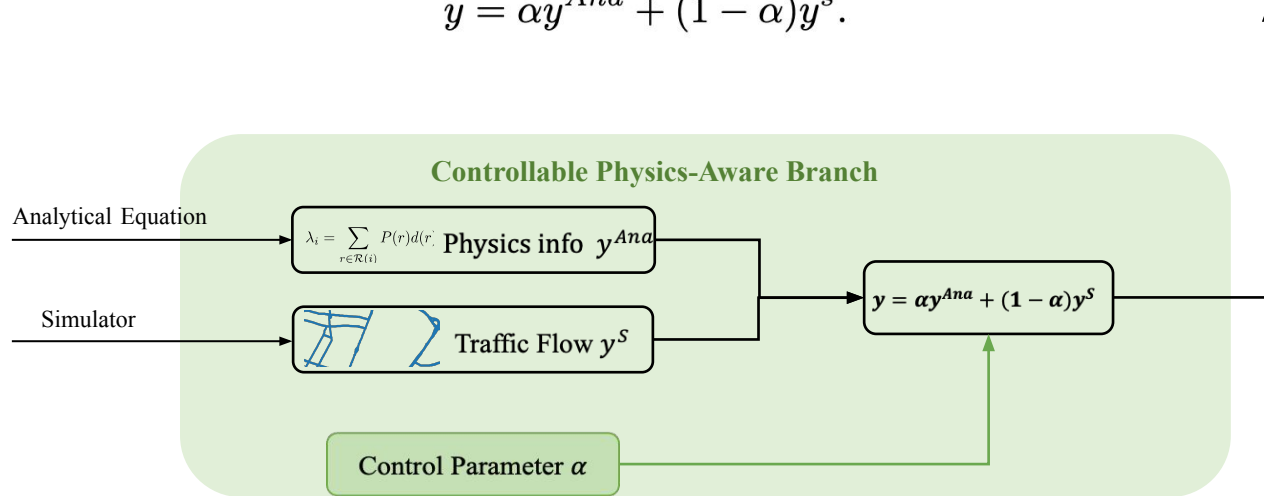
[1] Osorio, Carolina. "High-dimensional offline origin-destination (OD) demand calibration for stochastic traffic simulators of large-scale road networks." *Transportation Research Part B: Methodological* 124 (2019): 18-43.



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- Balance between physics-knowledge and simulation information:

$$y = \alpha y^{Ana} + (1 - \alpha) y^S.$$



# Training Objective of Control-VAE

- Conventional CVAE variational lower bound[1]:

$$L_{\text{CVAE}} = -\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|d,y)} [\log p_{\psi}(d|\mathbf{y}, \mathbf{z})] + D_{KL}(q_{\phi}(\mathbf{z}|d,y) || p_{\psi}(\mathbf{z}|\mathbf{y}))$$

- Regularizer aligning the physics information with simulator's behavior:

$$L_{\text{MSE}} = ||y^{\text{Ana}} - \tilde{y}||_2^2$$

- Total loss:

$$L = L_{\text{CVAE}} + \gamma L_{\text{MSE}}$$

[1] Sohn, Kihyuk, Honglak Lee, and Xinchen Yan. "Learning structured output representation using deep conditional generative models." *Advances in neural information processing systems* 28 (2015).





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# Experiments Setting

- High dimensional Munich network[1]:
  - 5329 origin-destination (OD) pairs;
  - 507 detector locations;
  - 5:00 am -10:00 am;
  - $x_c = (p + q \times \delta) \times \hat{d}$ , p: reduction; q: randomization
    - Set I: p=0.7, q=0.15;
    - Set II: p=0.7, q=0.3.
- Simulator:
  - Simulation of Urban MObility (SUMO);
  - Other traffic simulators ...

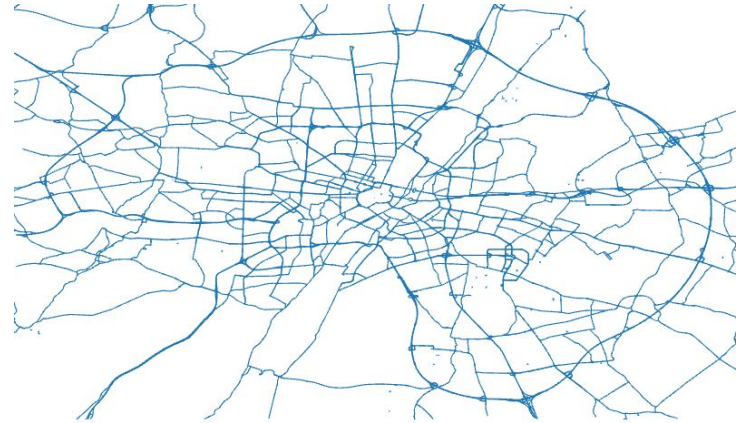


Figure 5: Overview of Munich Traffic Network



[1] Qurashi, Moeid, et al. "Dynamic demand estimation on large scale networks using Principal Component Analysis: The case of non-existent or irrelevant historical estimates." *Transportation Research Part C: Emerging Technologies* 136 (2022): 103504

# Experiments Results - Comparison to SOTA Methods

Table 1: Comparison to the SOTAs on the Munich Network (RMSN (%)).

		Munich 5-6	Munich 6-7	Munich 7-8	Munich 8-9	Munich 9-10
Set I	SPSA	24.67±1.81	24.59±2.41	21.24±1.33	47.06±0.64	18.40±0.44
	PC-SPSA	<b>15.40±2.71</b>	35.05±0.44	22.64±2.61	28.36±4.66	21.94±0.51
	<b>ControlVAE</b>	17.40±0.87	<b>22.02±1.45</b>	<b>17.55±1.29</b>	<b>19.89±1.84</b>	<b>16.28±1.22</b>
Set II	SPSA	18.00±1.12	43.10±0.24	55.89±2.31	50.04±0.61	36.13±0.43
	PC-SPSA	15.03±0.71	35.66±0.42	23.46±3.23	28.79±0.51	22.31±0.51
	<b>ControlVAE</b>	<b>14.89±0.56</b>	<b>21.74±1.59</b>	<b>18.32±1.83</b>	<b>21.02±1.84</b>	<b>16.38±1.02</b>

- The higher calibration quality of the data-driven approach for high-dimensional problems than traditional state-of-the-art methods (SPSA, PC-SPSA) .



# Experiments Results - Comparison to Neural Baselines

Table 2: Comparison to Neural Baselines on the Munich Network (RMSN (%)).

		Munich 5-6	Munich 6-7	Munich 7-8	Munich 8-9	Munich 9-10
Set I	Original	97.08±1.23	52.20±0.72	36.40±1.63	49.76±0.80	43.92±0.56
	CVAE	22.00±1.59	22.98±2.15	19.28±2.81	23.31±2.81	30.56±1.09
	CVAE-catt	18.45±0.61	22.41±2.00	19.33±1.70	20.96±0.82	16.34±1.17
	CVAE-phy	21.43±2.85	22.04±2.48	20.78±1.35	23.92±2.37	17.48±1.30
	<b>ControlVAE</b>	<b>17.40±0.87</b>	<b>22.02±1.45</b>	<b>17.55±1.29</b>	<b>19.89±1.84</b>	<b>16.28±1.22</b>
Set II	Original	97.20±1.62	87.52±1.62	101.3±3.15	70.21±0.64	80.70±0.58
	CVAE	46.23±0.91	24.57±1.90	26.28±1.60	27.09±1.69	18.25±0.67
	CVAE-catt	16.43±0.79	30.72±1.76	18.47±1.75	21.60±1.55	19.42±1.15
	CVAE-phy	15.73±1.00	22.75±1.71	21.76±3.08	28.63±1.86	17.93±1.04
	<b>ControlVAE</b>	<b>14.89±0.56</b>	<b>21.74±1.59</b>	<b>18.32±1.83</b>	<b>21.02±1.84</b>	<b>16.38±1.02</b>

- Original: RMSE between the traffic flow generated by the noised OD demand and the real traffic flow.



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- Control VAE vs. others: demonstrate it can control the interaction of assistance between physics-informed and data-driven machine learning.



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- CVAE-catt: Cross-attention using simulation information;
- CVAE-phy: Cross-attention using analytic information.
  - Further improvement for efficiency.



# Qualitative Evaluations - Convergence speed

- SPSA and PC-SPSA can only be serially iterated, while generative neural network-based methods can collect data **parallelly**.
- The generative neural network-based approach uses **fewer samples** to achieve the desired performance, than both SPSA and PC-SPSA, where people generally care about the best result across all current iterations.

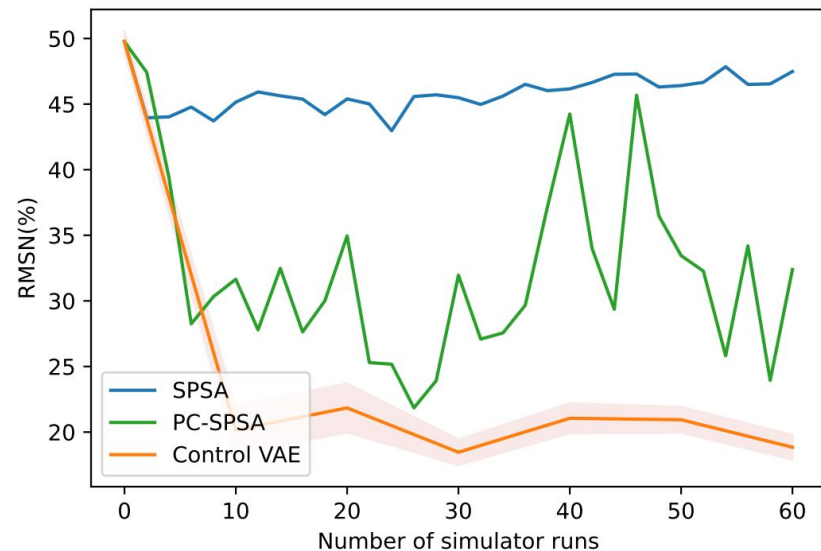


Figure 6: The converge curve based on the count of simulator running on M89, Set I.



# Qualitative Evaluations - OD Distribution

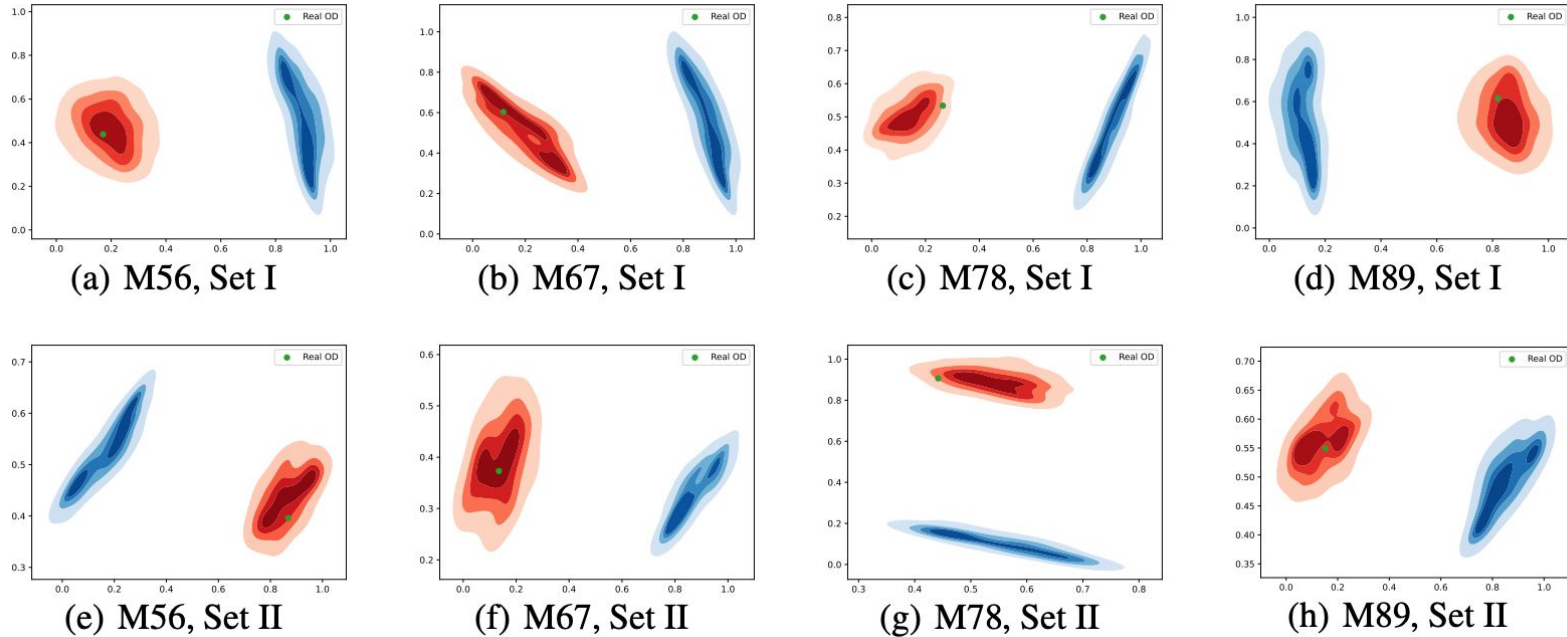
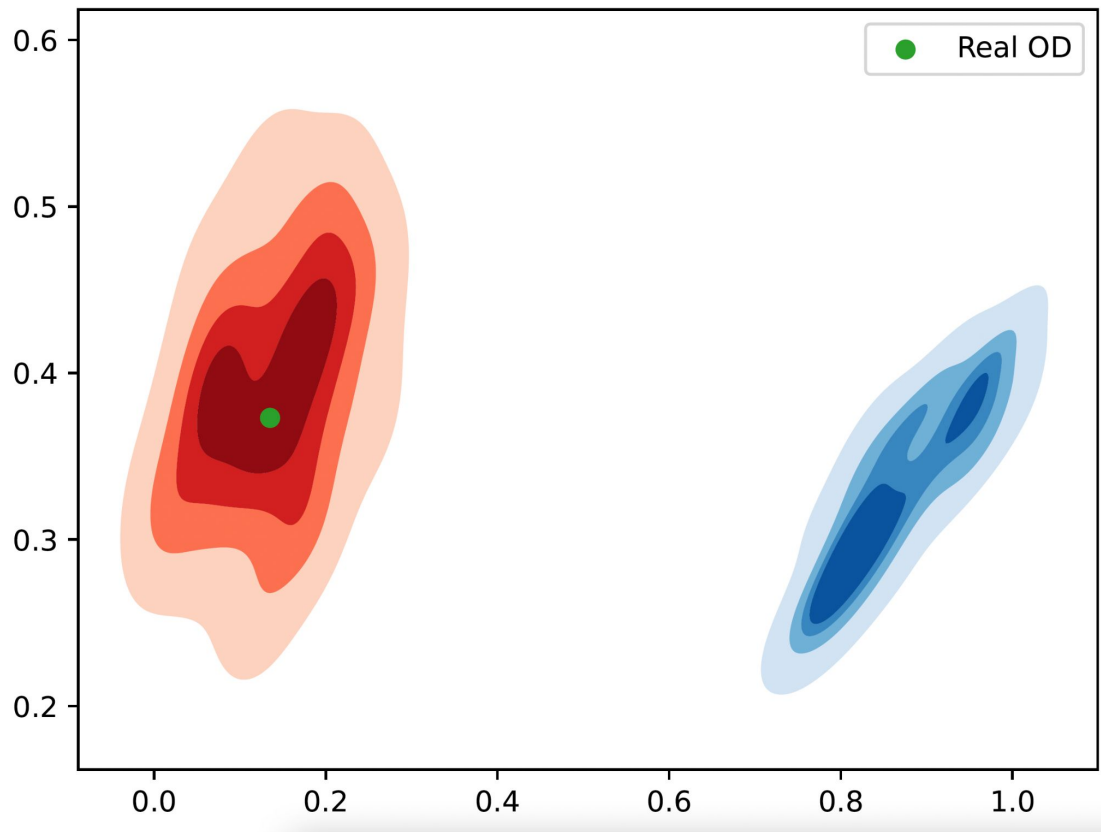


Figure 7: Calibration Distribution Results on OD demand. The **blue cluster** is the prior distribution of input OD and the **red cluster** is the calibrated OD distribution condition on observed traffic counts. The **green dot** refers to the real OD that we aim to identify.





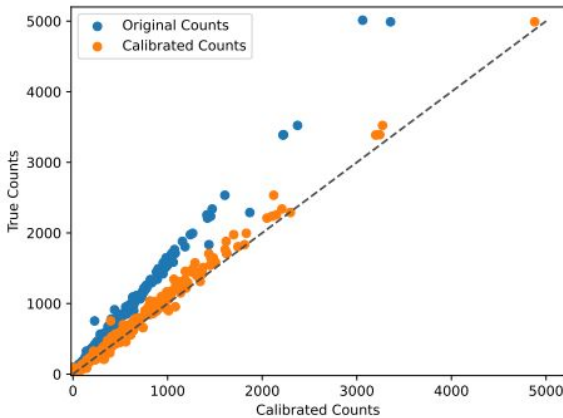
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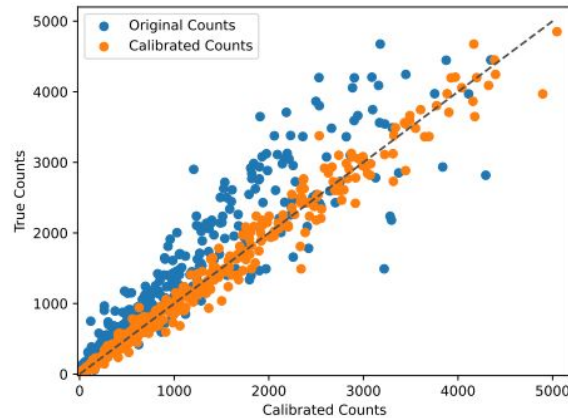
- The effectiveness of our model on large-scale datasets to assign high probability to the true OD solution;
- Our proposed model assigns some probability to other possible solutions, not just the single most likely one.



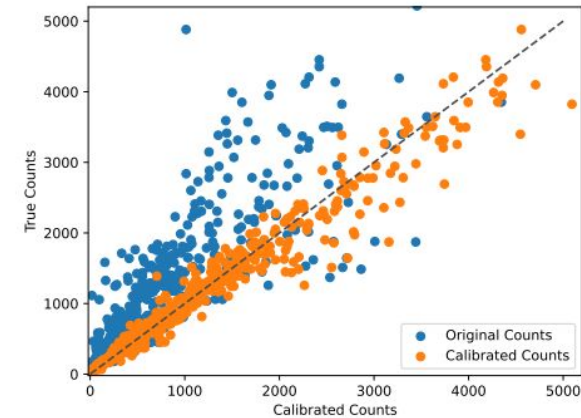
# Qualitative Evaluations - Traffic Flow Calibration



(a) M56, Set I



(b) M67, Set I



(c) M89, Set I

Figure 8: Calibration Results on Traffic Counts.

- The proposed method identifies ODs that yield an excellent fit to the real data (even for the morning peak period of 8:00 am-9:00 am).
- This excellent fit holds for all detector locations.





# Controllable Physics-Aware Generative Model for Urban Travel Demand Calibration



# Thank you!

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